Hall, G. (2014). Integrating real-world numeracy applications and modelling into vocational courses.
*Adults Learning Mathematics: An International Journal, 9*(1), 53-67

# Integrating Real-World Numeracy Applications and Modelling into Vocational Courses

**Graham Hall**

Coleg Meirion-Dwyfor, Dolgellau, United Kingdom, LL40 2SW

<graham.hall@gllm.ac.uk>

## Abstract

Practitioner research is in progress at a Further Education college to improve the motivation of vocational students for numeracy and problem solving. A framework proposed by Tang, Sui, & Wang (2003) has been adapted for use in courses. Five levels are identified for embedding numeracy applications and modelling into vocational studies: Extension; Special Subject; Investigation Report; Paper Discussion; and Mini Scientific Research. These levels represent a progression from applications set by the teacher, through increasing student involvement in the solution of real world problems, to totally independent project work. Case studies are presented of the incorporation of the five levels of application in engineering, construction, computing, and environmental science courses. In addition to student motivation, teaching staff observed that improvements have occurred in: use of specialised mathematical vocabulary; the combined use of numerical and algebraic methods in problem solving; and abstract reasoning, and a deeper level of understanding of the mathematics used in problem solving. A difficulty which has not yet been fully resolved is the reconciliation of a problem solving and project based approach to numeracy, and the requirement by some Examination Board numeracy syllabuses to assess specific mathematical methods.

Key words: numeracy, vocational education, modelling, applications, assessment

## Introduction

This paper describes practitioner research which is being carried out by tutors of vocational courses at a Further Education College in Wales. Students often begin vocational courses with a poor experience of school mathematics and lack enthusiasm to improve their mathematical skills. However, they will need to develop numeracy and problem solving as an essential aspect of their vocational training, for example: in subjects such as engineering or construction. The aim of the current project is to develop a framework of learning strategies which will interest and motivate students. It is hoped to develop students' numeracy skills within their vocational areas; and to help them to gain transferrable skills in critical thinking, creativity, teamwork and collaboration, and learning self-direction.

School mathematics in Britain, as in many other countries, is designed around a bottom-up academic model. Pupils learn mathematical methods within distinct topic areas such as: number, algebra and geometry, then work on example applications still within these same topic areas. The intention of the developers of mathematics syllabuses seems that pupils will progress to study subjects at an advanced level, such as sciences, where they will be able to make good use of the mathematical techniques they have learned. Figure 1 highlights the components of this bottom up academic model in Britain.

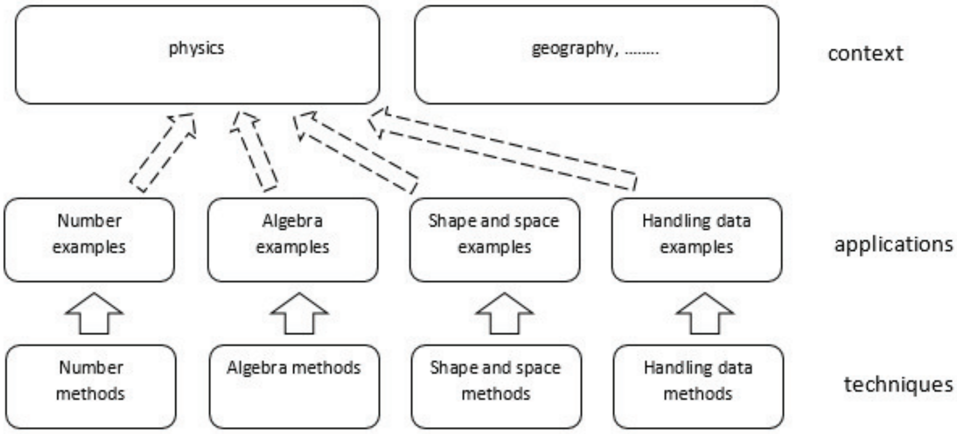



*Figure 1.* Bottom-up academic mathematics model

This model can present problems for students who leave school at the age of 16 to study a practical vocational course. They may view mathematics as a series of unrelated topics, some of which seem to have no relevance to their chosen profession. Algebra, in particular, is seen by many school leavers as having very little practical everyday.

## Methodology

Students entering further education courses in engineering, construction, computing, and environmental science at post-16 age took part in questionnaire surveys. This allowed the researchers to better appreciate and understand the attitudes and abilities in numeracy developed by the students during their school education.

Clinical interviews (Ginsburg, 1981) were then carried out with a total of 12 students chosen from the range of courses. The students were asked to give a commentary on their reasoning whilst attempting to solve various mathematical problems. From an analysis of the interview transcripts, four particular difficulties were identified:

- Lack of specialised mathematical vocabulary. Students had difficulty describing features of graphs, equations and other mathematical entities.
- No strong connection between number and algebra in problem solving (Lee & Wheeler, 1989). Students made no attempt to understand relationships in formulae by substituting numerical values, and made no attempt to devise formulae to simplify the repetitive handling of numerical data.
- A preference for justification by concrete example. Students generally preferred to use manipulation and measurement of solid shapes to solve problems, rather than abstract mathematical reasoning.

- Misuse of standard algorithms which had been learned in a superficial manner without full understanding. Examples causing difficulty included formulae for areas and volumes, sides of triangles, and trigonometry.

It became evident that there was little to be gained by continuing to teach in a way which had already been unsuccessful for some students. A new approach was therefore attempted by teaching staff participating in the project, and forms the basis of this paper. Thought was given to the development of open ended and ill-defined numeracy problems to be presented to students which would realistically simulate work related tasks within the within their vocational areas.

A useful framework for introducing real world problems into mathematics teaching has been proposed by Tang, Sui, & Wang (2003) from work in China. Practitioner research during the current project has focussed on ways in which this framework could be successfully adapted for introduction into vocational courses at further education level. Five approaches are identified by Tang et al. for embedding numeracy applications and modelling: Extension; Special Subject; Investigation Report; Paper Discussion; and Mini Scientific Research. These approaches represent a progression from applications set by the teacher, through increasing student involvement in the solution of real world problems, to totally independent project work designed by students themselves.

At the end of an academic year, students were re-interviewed to investigate changes in their perceptions of numeracy, and to investigate the extent to which their abilities in problem solving had developed.

## Applying Numeracy

A distinction is made by a number of researchers (Hoyles et al., 2000; Dingwall, 2000; Coben, 2000) between mathematics, which is taken to be a set of quantitative methods, and numeracy which has wider links with the real world. Numeracy need not be at an elementary level but might include, for example, the advanced mathematics used by engineers or scientists. Numeracy requires knowledge of the real world context in which the problem occurs. It is essentially a practical problem solving activity drawing upon appropriate mathematical techniques, and the results obtained often need to be communicated to others in a way which is useful for decision making. The relationship between mathematics and numeracy developed by this model is illustrated in Figure 2.

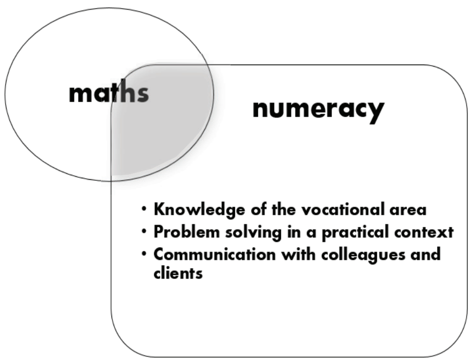



*Figure 2.* Relationship between mathematics and numeracy

The approach taken by teaching staff during the current project is to work downwards within vocational area to identify numeracy tasks undertaken by practitioners in everyday work. Tasks are analysed in collaboration with students, and solved using mathematical methods which might be familiar or which might need to be learned at this stage. Additionally, the work provides opportunities for consolidating mathematical knowledge in broader topic areas. This approach is illustrated in Figure 3.

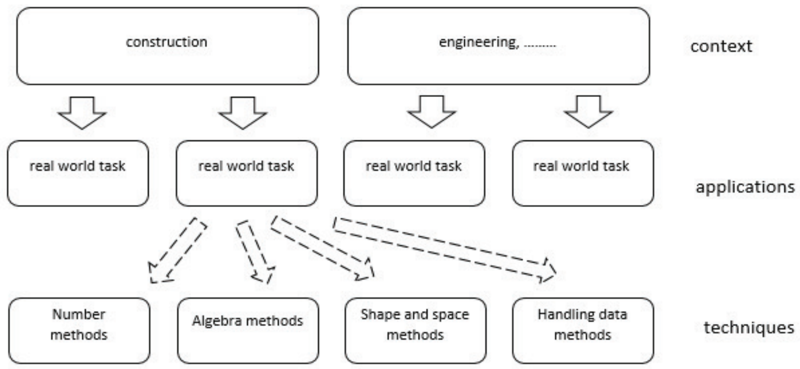



*Figure 3*. Top-down vocational numeracy model

Central to the numeracy approach which we are developing with our students is the MeE motivation model of Martin (2002) and Munns & Martin (2005), summarised in Figure 4:

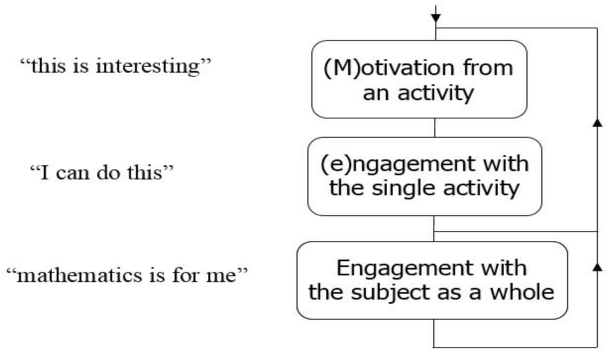



*Figure 4*. MeE motivation model of Munns & Martin (2005)

The model focuses on the need to motivate students by presenting interesting learning activities, and the self-satisfaction that students can gain from engaging successfully with these activities. This can lead students to develop a personal engagement with the subject as a whole, through the enjoyment and sense of achievement which it provides, so that they become intrinsically motivated to develop skills and knowledge to a higher level.

Munns and Martin advocate the introduction of the most interesting work from the very start of a course, as a means of generating enthusiasm. Teachers may need to simplify tasks to ensure that students achieve a successful outcome and gain a sense of achievement. It is important that the students consider the tasks to be realistic, relevant and worthwhile.

During the current research activities, teaching staff realised that a number of interesting and motivating tasks might need to be presented, but they hoped that individual students would reach the point of engaging with the subject as a whole. From this stage on, the work of the teacher would become much easier. The value of the subject would be clear to the students and they would be motivated to extend their knowledge and skills through independent learning.

## Naturally Occurring Numeracy

In a number of vocational areas, numeracy tasks occur quite naturally in everyday work. Two examples produced by colleagues are (a) curved work in carpentry, and (b) expedition planning, which are presented here:

### Curved work in carpentry (Slaney, 2013)

Amongst the more advanced practical skills taught to carpentry students are methods for constructing curved door and window frames of various designs. Designs have to be produced as a bench template for cutting the timber components.

A challenge presented to students was to construct a gate in the form of a Tudor Arch, which traditionally has the geometrical design illustrated in Figure 5:

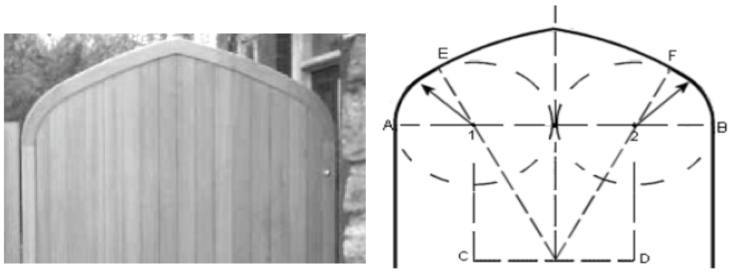



*Figure 5.* Geometrical construction method for a Tudor arch

Students investigated the construction method:

- Two circles of equal radius are drawn to set the width AB of the arch. Two arcs AE and BF of these circles form parts of the completed arch.
- A square is constructed, with the distance between the circle centres 1 and 2 as the length of each side,
- The mid-point of the bottom edge of the square, CD, is used as a centre for constructing the upper arcs between E and F to complete the arch.

The group were interested to investigate the geometry of other traditional arch designs developed by masons and carpenters during different historical periods.

### Expedition planning

Students who are training to become outdoor pursuits instructors are required to make reasonably accurate estimates of the time which expeditions will take over mountainous terrain as part of the procedure for safety planning. A mathematical formula known as Naismith's Rule can be used for estimating journey time. This determines a time based on walking speed over flat ground, then adds extra time for the amount of ascent and descent necessary during the journey.

Students using Naismith's Rule have found that the time calculations for expeditions in the mountainous area of North Wales are very inaccurate. This is due to wide variations in the time taken to cross different types of terrain. Walking speed is much slower across moorland, overgrown forest or wetland than along well constructed footpaths. Scrambling over rocks on mountainsides is particularly slow.

As a project, students have documented the actual times taken for the different stages of a number of expedition routes, and have related these to the nature of the terrain. They are attempting to develop a more accurate journey time formula to improve on Naismith's Rule. This project is a good example of the application of the modelling cycle of Blum and Leiß (Keune & Henning, 2003).

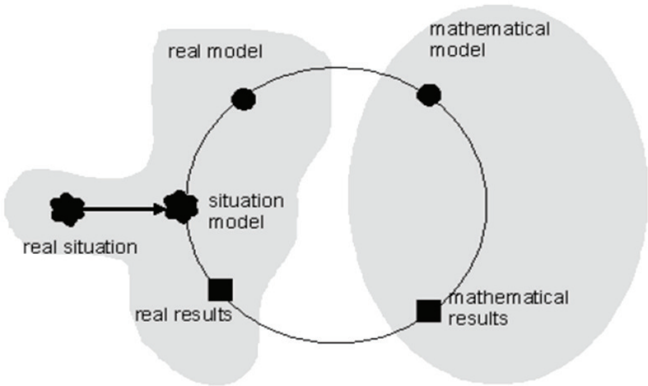



*Figure 6.* The modelling cycle of Blum and Leiß

The modelling cycle considers the manner in which a mathematical modelling problem can be conceptually divided into two domains – the real world domain, shown on the left in Figure 6, and the mathematical domain shown on the right.

The modelling cycle begins in the real world domain, where it is necessary to identify the factors which are important to the outcome of the model. The relationships between these factors are then assessed in descriptive terms. Modelling then moves to the mathematical domain, where the factors are expressed in terms of a formula and numerical examples are run to generate modelling predictions. These modelling predictions are then related back to the real world domain and checked against actual observations. If necessary, the modelling assumptions can be revised and the model re-run, until an acceptable solution is found.

A particular value of this project has been to help students make a connection between algebra and number, with these mathematical methods employed together effectively in problem solving. Students often have difficulty in constructing algebraic formulae from theoretical relationships between variables, or from the identification of patterns in observed data. We have found that the plotting of graphs can provide a helpful conceptual link between number and algebra.

## Framework for Numeracy and Applications based on the work of Tang, Sui & Wang (2003)

It is valuable for numeracy tutors to make use of naturally occurring numeracy tasks, such as the carpentry work mentioned above which forms an essential component of the course syllabus. However, it is sometimes necessary for tutors to develop additional applications to broaden the mathematical and problem solving skills of vocational students. Many topics studied on vocational courses can, with imagination on the part of the tutor, provide opportunities for interesting and realistic numeracy problem solving.

Tang, Sui & Wang (2003) proposed a framework which we have adapted for use in vocational courses during this project. The original framework was intended to provide a practical structure in which mathematics students could apply their mathematical skills in realistic real world situations. Our approach is somewhat different, in that we have used the framework as a structure by which vocational students might investigate real world problems through the application of numeracy. Whilst the students of Tang et al. would be experienced in mathematical techniques but perhaps unfamiliar with their applications in the real world, our students would be familiar with the types of problems arising in vocational situations but might need to develop further mathematical skills to solve these. The overall aim in both cases is to develop practical numeracy problem solving skills, though from different starting points.

Five levels were identified by Tang et al. for incorporating applications and modelling into mathematics courses: (a) Extension, (b) Special Subject, (c) Investigation Report, (d) Paper Discussion, and (e) Mini Scientific Research. Examples of tasks illustrating each of these levels are presented below. The five levels represent a progression from applications set by the teacher, through increasing student involvement in the solution of real world problems, to totally independent project work designed by students themselves.

## Extension

In this approach, students who have been studying a mathematical topic are presented with an ill-defined real world problem where they need to seek out additional data for its solution. As an example, consider the problem in Figure 7 which might be given at the end of a study of trigonometry. The problem cannot be solved without obtaining measurements.

A photographer is intending to travel on the London Eye, a large Ferris wheel in London. She wishes to take panoramic views across the city, but needs to be at least higher than the roof of the nearby County Hall building to do this. She would like to know how many minutes will be available for the photographic session.

*Figure* 7. Example of an ill defined problem

In this case, the student should obtain actual data, or at least reasonable estimates, for the speed of rotation of the wheel, its diameter, and the height of the adjacent building. This might be found by use of the Internet. The student is then free to devise his/her own method for numerical, graphical or analytic solution of the problem.

An example solution produced by a student is shown in Figure 8. Trigonometry has been used to obtain a formula linking the height of a car above the ground, H, to the diameter of the wheel, the height of the central axel above the ground, and the angle of rotation θ. A graph was then plotted using a spreadsheet, and an estimate made of the time during which the car was above the height of the nearby building.

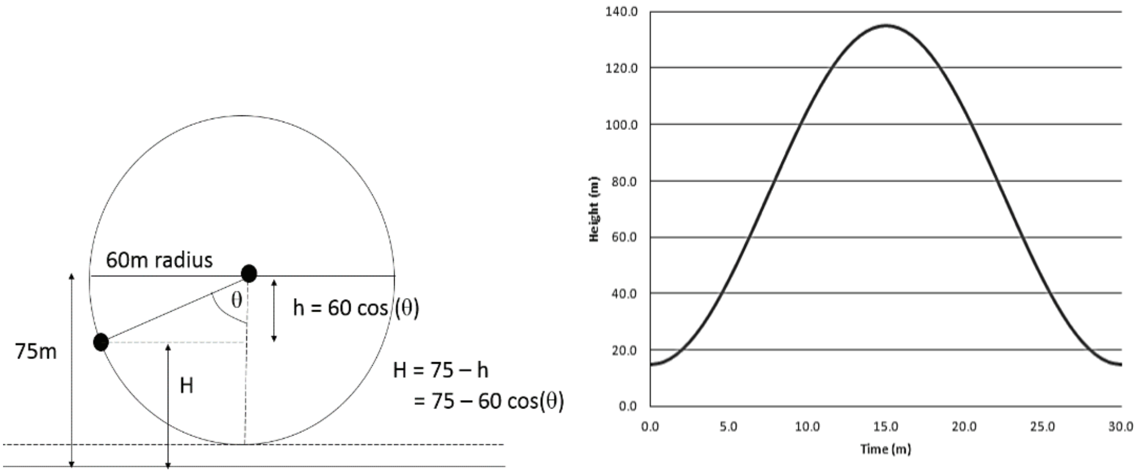



*Figure* 8. A possible graphical solution of the London Eye problem. The wheel has a diameter of 120m, and rotates in 30 minutes.

## Special Subject

Students who have studied a vocational topic were given the opportunity to investigate the topic further through a quantitative project. This approach was used successfully with construction students who had been studying heat losses from buildings. After discussion of the insulating properties of different building components, students developed their own spreadsheets to determine the heat losses from a house. This allowed investigation of the effects of double glazing of windows, cavity insulation of walls, and insulation of the roof space, and gave a deeper understanding of the mathematics involved.

The model makes use of the dimensions of each wall of a room, and its construction material, to estimate heat losses. This heat loss is also dependent on the average temperature difference between the two sides of the wall. Heat losses are added for the floor and ceiling, to obtain total heat losses for the room. Allowance must also be made for heat loss through air ventilation in the room.

| C | D | E | F | G | H | I | J | K |
|---|---|---|---|---|---|---|---|---|
| | | | House heating calculations | | | | | |
| | | length | height | width | area | inside temp | outside temp | temp diff |
| wall1 | wall | 14 | 8 | | 72 | 70 | 30 | 40 |
| Back wall | window | 10 | 4 | | 40 | 70 | 30 | 40 |
| | door | | 8 | | 0 | 70 | 30 | 40 |
| wall2 | wall | 17 | 8 | | 136 | 70 | 65 | 5 |
| Party wall | window | | 8 | | 0 | 70 | 65 | 5 |
| | door | | 8 | | 0 | 70 | 65 | 5 |
| wall3 | wall | 14 | 8 | | 112 | 70 | 70 | 0 |
| Lounge wall | window | | 8 | | 0 | 70 | 70 | 0 |
| | door | | 8 | | 0 | 70 | 70 | 0 |
| wall4 | wall | 14 | 8 | | 112 | 70 | 65 | 5 |
| Kitchen wall | window | | 8 | | 0 | 70 | 65 | 5 |
| | door | | 8 | | 0 | 70 | 65 | 5 |
| wall5 | wall | 3 | 8 | | 24 | 70 | 60 | 10 |
| Hall wall | window | | 8 | | 0 | 70 | 60 | 10 |
| | door | | 8 | | 0 | 70 | 60 | 10 |
| wall6 | wall | | 8 | | 0 | 70 | | 70 |
| | window | | 8 | | 0 | 70 | 0 | 70 |
| | door | | 8 | | 0 | 70 | 0 | 70 |
| floor | | 17 | | 14 | 238 | 70 | 30 | 40 |
| ceiling | | 17 | | 14 | 238 | 70 | 65 | 5 |
| | | | | | | | | |
| room volume | 1904 | 17 | 14 | 8 | | air changes per hour | | 2 |

*Figure 9.* House heat loss

## Investigation Report

For this approach, students gather their own primary data through surveys, laboratory or fieldwork measurements, then process the data using appropriate mathematical methods. In this way, it is hoped to gain a clearer interpretation of the data and to obtain insights which were not initially obvious.

As an example, geography students investigating coastal processes measured pebbles which were being transported along a shingle spit by wave action. Results and analysis are presented in Figure 10. The chart on the left plots the average dimension of pebbles against position along the beach. The shingle spit originates at a cliff line at Friog on the left of the chart, and extends out into the estuary for a distance of approximately 1.5km, past Fairbourne to Barmouth Ferry.

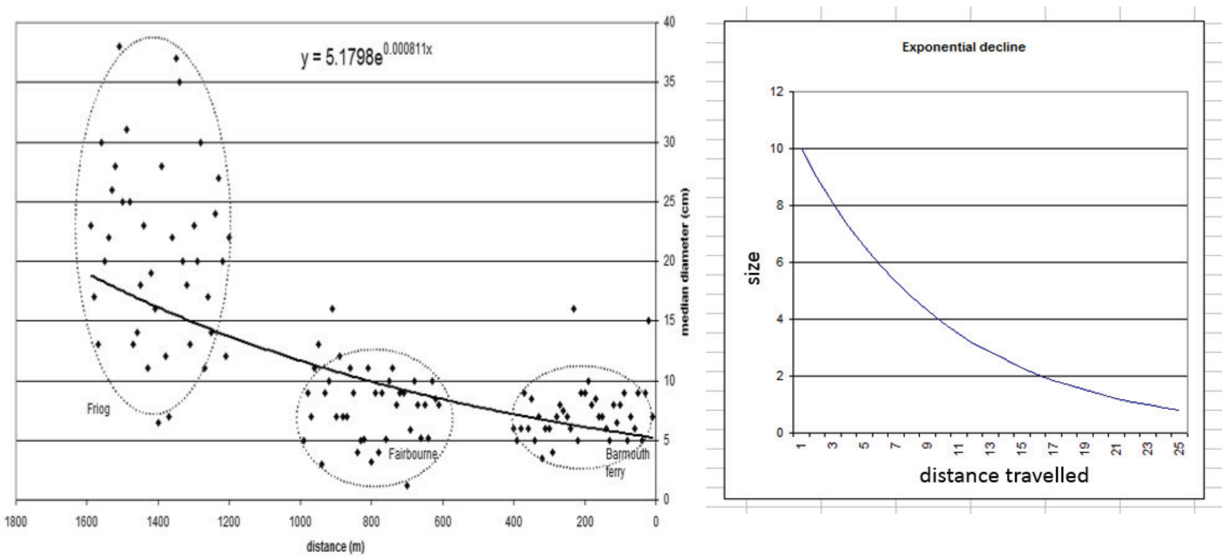



*Figure 10.* A comparison of field data and a theoretical model for pebble erosion

It was seen that although a mixture of pebble sizes was present at each location visited, there was a reduction in mean size during transport along the shingle spit. Discussion between students and their tutor lead to a hypothesis that the rate of size reduction would be proportional to the actual pebble size — large pebbles would be eroded more easily by wave action than small pebbles.

A spreadsheet numerical model was developed for a constant percentage reduction in size for each distance unit, leading to the familiar negative exponential curve. The theoretical curve produced in the spreadsheet model to the right was seen to closely reflect the best fit curve through the actual field data, supporting the initial hypothesis connecting rate of erosion directly to pebble size.

## Paper Discussion

The approach used here is to present students with an interesting and challenging vocational mathematics task, then provide resources from books, journal articles or the Internet which will allow the students to teach themselves the necessary quantitative techniques for solving the problem. This contrasts with the normal teaching approach in which the tutor provides instruction, and is intended to encourage students to develop as independent learners.

An example presented to computing students was to model an epidemic of a non-fatal illness such as influenza. Published articles were provided which explained the recurrence relations which form the Simple Epidemic Model (Keeling, 2001). The population is modelled as three groups:

- Susceptible: those who can catch the illness,
- Infected: those who have the illness, and could infect others, and
- Recovered: those who cannot catch the illness again and are no longer infectious to others.

In each time period, a number of people will catch the disease and move from the Susceptible to the Infected group. The number of persons infected will depend on the proportion of the population who are susceptible and infected, and come into contact. It will also depend on the infectiousness of the disease – a variable known as the Epidemiological Parameter.

In the time period, others will recover and move from the Infected to the Recovered group. This number will depend on the average time a patient takes to recover from the illness. Results from a run of the spreadsheet recurrence relation model are shown in Figure 11.

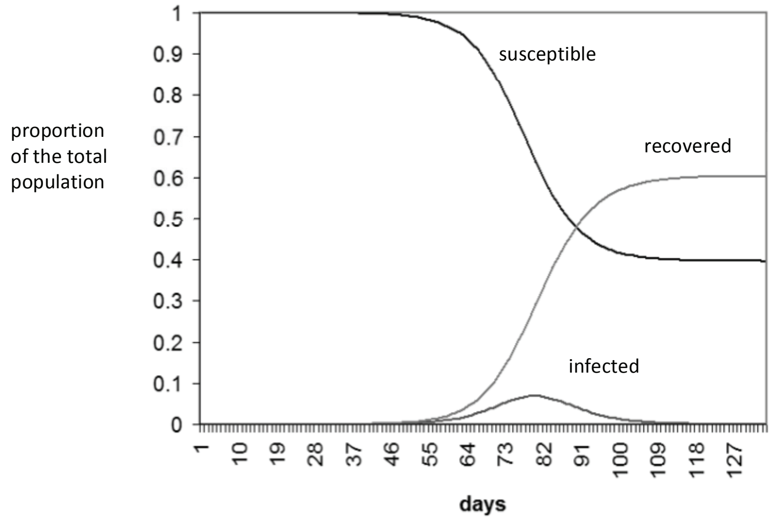



*Figure 11.* Results generated by students in running the Simple Epidemic model.

It is seen that the epidemic begins to decline when the number of recovered people in the population exceeds the number susceptible. This is due to the reducing likelihood of an infected person coming into contact with a susceptible person to spread the illness.

## Mini Scientific Research

This approach represents the maximum level of student involvement in the planning, investigation and analysis of data for a substantial numeracy project related to their vocational area. An example of a project carried out by engineering students has been the investigation of the motion of a car when passing over a speed hump, in response to the springs and shock absorbers of the car suspension system. Results from a run of the spreadsheet model are shown in Figure 12.

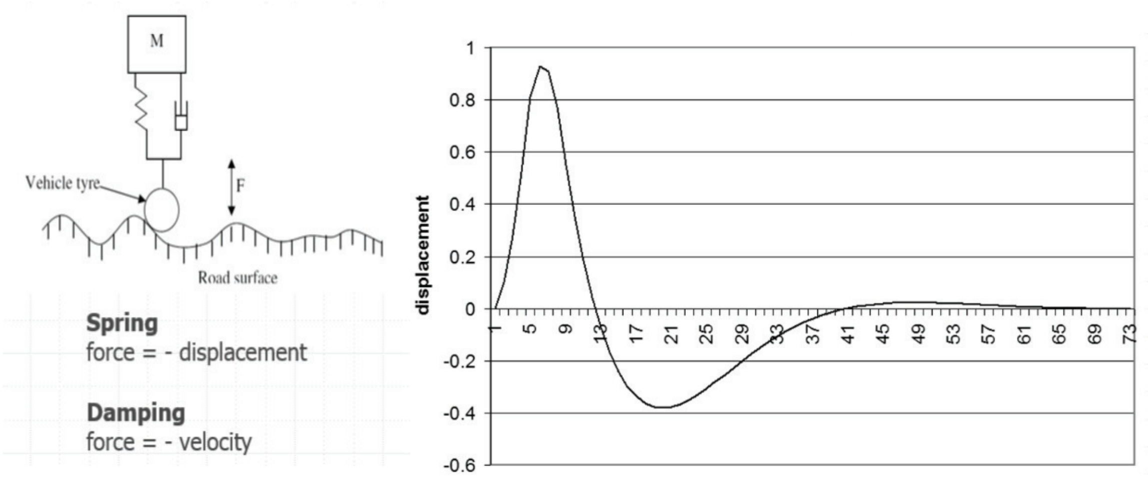



*Figure 12.* Damped simple harmonic model for the motion of a car passing over a speed hump

The model was developed by the students as a recurrence relation for small time intervals. At the start of the time interval, the current vertical velocity and acceleration of the car body were known. Initially these are zero for steady motion along the road.

As the car climbs the speed hump, it transmits a vertical force to the car body. In the case of the spring component, this is dependent on the shortening of the spring according to Hooke's Law. For the damper component, however, a reverse force is generated which is proportional to the vertical velocity of the damper piston. Vertical acceleration of the car body can then be modelled from the resultant force through the suspension system and the mass of the car body.

Students were able to compare the results of spreadsheet modelling with video film which they produced of the actual motion of cars passing over speed humps at different speeds.

## Evaluation

The researcher conducted observations of students who were undertaking numeracy tasks, and examined the solutions students produced before interviewing participants about their experiences during the project and their broader attitudes towards numeracy and mathematics. The project is ongoing, but it is clear that higher levels of interest and motivation have been generated by the various tasks, and students' confidence in using mathematical techniques has been improved. In particular, problems identified early in the year have been addressed to a significant extent:

- Use of specialised mathematical vocabulary is more evident,
- Numerical and algebraic methods were being combined in the solution of problems,
- Skills in abstract reasoning have improved, and
- A deeper level of understanding of the mathematics used in problem solving is evident.

A difficulty which has not yet been fully resolved is the reconciliation of a problem solving and project based approach to numeracy, and the requirement by some Examination Board syllabuses to assess specific mathematical methods. As an example, we might consider the Essential Skills Wales Application of Number qualification.

From the description:

> The aim of the Application of Number standards is to encourage candidates to develop and demonstrate their skills in using number to tackle a task, activity or problem by collecting and interpreting information involving numbers, carrying out calculations, interpreting results and presenting findings. (WJEC, 2013)

this qualification appears to closely embrace a real-world problem solving approach to numeracy. However, closer examination of the assessment requirements presented in Figure 13 shows that a large number of particular mathematical methods must be demonstrated in the work submitted by candidates.

It is evident from this list that no single realistic real-world project, or even small number of projects, is likely to come anywhere near covering all the stated requirements. There is therefore a tendency for tutors to revert to the bottom-up model of teaching mathematical topics individually to cover the syllabus, and vocational applications become limited and unconvincing to students.

A compromise approach is to primarily employ real-world problem solving as a means of motivating students, but also allocate time at the end of each project session to provide broader coverage of related mathematical methods and topics. Students are made aware that this is necessary in order to meet assessment requirements. For example, after solving a graphical problem which is discovered to involve an exponential function, students may be introduced to other related functions such as powers, inverse powers and logarithms. After the use of trigonometry to creatively solve an ill-defined circular motion problem, students might examine the use of trigonometric applications in other areas such as topographic surveying.

IN ORDER TO SHOW THAT YOU ARE COMPETENT, YOU NEED TO ………………

- use powers and roots
- use compound measures
- use mental arithmetic involving numbers, simple fractions, and percentages
- work out missing angles and sides in right-angled triangles from known sides and angles
- calculate with sums of money in different currencies
- calculate, measure, record and compare time in different formats
- estimate, measure and compare dimensions and quantities using metric and, where appropriate, imperial units, and check the accuracy of estimates
- calculate within and between systems and make accurate comparisons
- draw 2-D representations of simple 3-D objects
- solve problems involving irregular 2-D shapes
- work out actual dimensions from scale drawings and scale quantities up and down
- work out proportional change
- compare distributions, using measures of average and range, and estimate mean, median and range of grouped data

………………

*Figure 13.* Extract from the specification for Essential Skills Wales Application of Number

## Further Development

Creative problem solving can provide a structure for introducing new mathematical topics in a way which is motivating for students, demonstrates immediate relevance to vocational studies, and supports a deeper understanding of mathematical methods. As an example, a new approach has been used to introduce engineering students to calculus for the first time at the start of their course. Students are asked to estimate the volume of the centre cone of a jet engine (Fig.14):

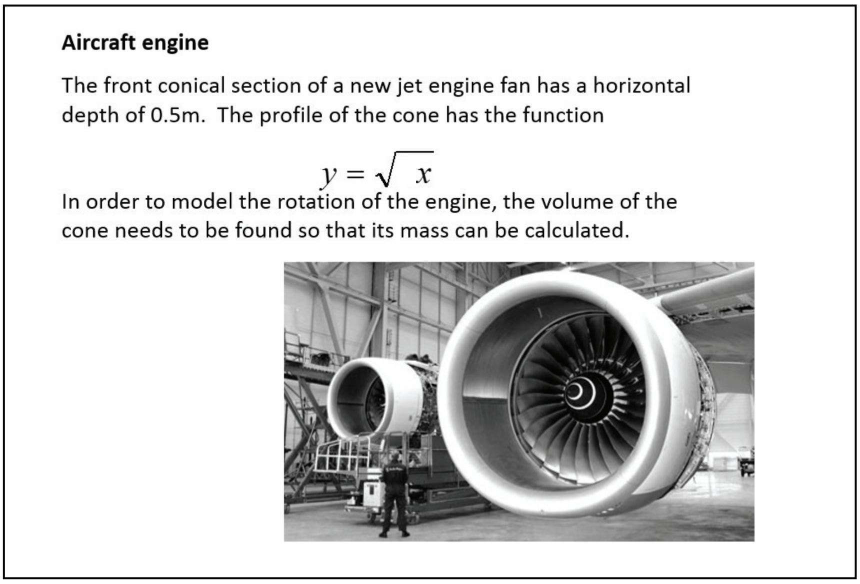
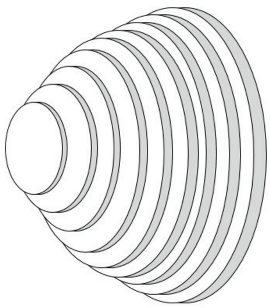



*Figure 14.* Volume calculation problem

In discussions between the tutor and the student group, it was agreed that the volume could be estimated by dividing the cone into a series of cylinders, with volumes then calculated and totalled using a spreadsheet (fig.14):



$$\int_0^{0.5} \pi . r^2 .dx$$

$$= \int_0^{0.5} \pi \left(\sqrt{x}\right)^2 .dx$$

$$= \pi \left[ \frac{1}{2} x^2 \right]_0^{0.5} = 0.125\pi$$

volume = $\sum \pi r^2 t$

Figure 15. *Solution to the volume calculation problem*

Students were readily aware that the accuracy of the estimate would increase as the number of cylinders was increased, although in practice the number of cylinders would be limited by the capacity of the spreadsheet program. Integration was then introduced as an alternative quick and easy method of finding the total volume of an infinite number of infinitely thin cylinders – effectively providing the exact answer to the problem. The group were suitably impressed by the power of mathematics, and the effort that can be saved by applying appropriate mathematical methods.

Overall, the development of numeracy through problem solving in vocational areas, either by naturally occurring applications or use of the framework of Tang et al. (2003), is seen as an effective way of increasing student motivation and creativity.

## Conclusion

This project has examined approaches to improving the numeracy and problem solving skills of students in engineering, construction, computing, and environmental science courses at a further education college. It has been evident to the teaching staff that student motivation is critical to developing numeracy. Principal factors affecting student motivation were found to be: the relevance of numeracy tasks to students' main courses, the realism and authenticity of tasks in the relevant vocational field, and the intrinsic interest and challenge of the problems presented.

A definite advantage of the use of open ended and ill-defined problem solving activities, apart from improving student motivation, was to encourage the development of wider numeracy skills. These include: communication of mathematical ideas in a form suitable for decision making, use of practical and common-sense techniques in obtaining solutions to problems, skills in data collection, and estimation of results to appropriate levels of accuracy. A particularly pleasing aspect observed by the researchers was an improved ability by students to inter-relate the numerical, graphical and algebraic representations of data sets.

The framework introduced by Tang et al. (2003) was considered by the teaching staff to provide a range of interesting opportunities for planning and structuring student activities which could be integrated into vocational courses. The student activities proposed by Tang et al. were seen to encourage: problem solving, group co-operative working, and independent learning.

## Acknowledgements

Grateful thanks are due to numeracy tutors and students at Coleg Meirion-Dwyfor, Dolgellau, for allowing their work to be observed, and to college managers for allowing and encouraging practitioner research. Any opinions, findings, conclusions or recommendations are those of the author and do not necessarily reflect the views of Coleg Meirion-Dwyfor.

## References

Coben, D. (2000). Numeracy, mathematics and adult learning. In I. Gal (Ed.), *Adult Numeracy Development: Theory, research, practice* (pp. 33-50). Cresskill, NJ: Hampton Press.

Dingwall, J. (2000). *Improving numeracy in Canada*. Office of Literacy and Essential Skills, Canada.

Ginsburg, H. (1981). The clinical interview in psychological research on mathematical thinking: Aims, rationales, techniques. *For the learning of mathematics*, 4-11.

Hoyles, C., Wolf, A., Molyneux-Hodgson, S. & Kent, P. (2000) *Mathematical Skills in the Workplace*. Final Report, University of London, Science, Technology and Mathematics Council, London.

Keeling, M. (2001). The mathematics of diseases. *Plus Magazine: Living Mathematics* 44. Retrieved from http://plus.maths.org/content/os/issue14/features/diseases/index

Keune, M. & Henning H. (2003). Modelling and Spreadsheet Calculation. In Qi-Xiao Ye, W. Blum, K. Houston & Q.-Y. Jiang (Eds), *Mathematical Modelling in Education and Culture: ICTMA 10* (pp. 233–248). Chichester, UK: Horwood Publishing.

Lee, L., & Wheeler, D. (1989). The arithmetic connection. *Educational Studies in Mathematics*, *20*(1), 41-54.

Martin, A. (2002). Motivation and academic resilience: Developing a model for student enhancement. *Australian journal of education*, *46*(1), 34-49.

Munns, G. & Martin, A. (2005). It's all about MeE: A motivation and engagement framework. In *Australian Association for Research in Education Conference, University of Western Sydney.* Retrieved from http://www. aare. edu. au/05pap/mun05400. pdf.

Slaney, S. (2013). *A Study of Teaching Methods to Increase the Motivation for Numeracy amongst Vocational Students*. Unpublished BA Dissertation. UK: Newport University.

Tang, A., Sui, L. & Wang, X. (2003). Teaching patterns of mathematical application and modelling in high school. In Qi-Xiao Ye, W. Blum, K. Houston & Q.-Y. Jiang (Eds.), *Mathematical modelling in education and culture: ICTMA 10* (pp. 233–248). Chichester, UK: Horwood Publishing.

Welsh Joint Education Committee (2013) Application of number - Essential Skills Wales. WJEC. Retrieved from http://www.wjec.co.uk/index.php?subject=184.